

## Chapter 8.1 part 2

Th 8.5 Lagrange's theorem

Let  $G$  be a finite group, and let  $K \subset G$  be a subgroup of  $G$ .

Then  $|G| = |K| [G : K]$

Cor 8.6 Let  $G$  be a finite group.

(1) For any  $a \in G$

$$|a| \mid |G|$$

(2) If  $|G| = k$ , then, for any  $a \in G$ ,

$a^k = e$  - the identity element in  $G$ .

Pf (1)  $\langle a \rangle$  is a subgroup of  $G$ , thus  $\frac{|\langle a \rangle|}{|G|}$

In addition,  $|a| = |\langle a \rangle|$

(2)  $k = |a| \cdot t$ ,  $t \in \mathbb{Z}$

$$a^k = a^{|a| \cdot t} = (a^{|a|})^t = e^t = e$$

Th 8.7 If  $|G| = p$ , a prime number, then  
the group  $G$  is cyclic (of order  $p$ ).

In particular, the order  
of any subgroup divides  
the order of the group

Pf Pick  $a \in G$ ,  $a \neq e$ . We have  $\langle a \rangle$  - cyclic subgroup of  $G$

$$|\langle a \rangle| / |G| = p$$

That implies  $|\langle a \rangle| = p$  meaning  $\langle a \rangle$  cannot be a proper subgroup.

$$\underline{\langle a \rangle = G}$$

Thus  $G$  is cyclic (in particular, abelian as any cyclic group is abelian)

Reu  $|\mathfrak{S}_3| = 3! = 6$

$\mathfrak{S}_3$  is not abelian.

Th 8.8 If  $|G|=4$ , then

either  $G \cong \mathbb{Z}_4$  or  $G \cong \mathbb{Z}_2 \times \mathbb{Z}_2$

(Pf in the book)

Th 8.9 If  $|G|=6$ , then

either  $G \cong \mathbb{Z}_6 \cong \mathbb{Z}_2 \times \mathbb{Z}_3$  or  $G \cong \mathfrak{S}_3$  (Pf in the book  
- lengthy analysis)

Classification we can do so far

$ G $	$G$
2	$\mathbb{Z}_2$
3	$\mathbb{Z}_3$
4	$\mathbb{Z}_4$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$
5	$\mathbb{Z}_5$
6	$\mathbb{Z}_6$ or $S_3$
7	$\mathbb{Z}_7$

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Zoo follows